IIT JAM : MATHEMATICS - STATISTICS

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections **A, B,** and **C**. All sections are compulsory. Question in each section are of different types.

2. **Section - A** contains a total of 30 **Multiple Choice Questions (MCQs)**. Each MCQ type question has four choices out of which only **one** choice is the correct answer. Question Q.1 - Q.30 belong to this section and carry a total of 50 marks. Q.1 - Q.10 carry 1 mark each and Questions Q.11 - Q. 30 carry 2 marks each.

3. **Section - B** contains a total of **10 Multiple Select Question (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 - Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section - C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 - Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Question Q.51 - Q.60 carry 2 marks each.

5. In all section, questions not attempted will result in zero mark. In **Section - A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks question, 2/3 marks will be deducted for each wrong answer. In **Section - B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section - C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

Special Instructions / Useful Data	
$\mathbb{R}$	The set of real numbers
$\mathbb{R}^n$	$\{(x_1, x_2, \ldots, x_n) : x_i \in \mathbb{R},\ i = 1, 2, \ldots, n\}$
det(M)	Determinant of a matrix M
I_n	Identity matrix of order $n \times n$, $n = 2, 3, \ldots$
g'	First derivative of a real valued function g
g''	Second derivative of a real valued function g
F^c	Complement of an event F
P(F)	Probability of an event F
P(F|G)	Conditional probability of an event F given the occurrence of event G
$X \sim f$	The probability density/mass function of the random variable X is f
E(X)	Expectation of a random variable X
Var(X)	Variance of a random variable X
$U(a, b)$	Continuous uniform distribution on the interval (a, b) $-\infty < a < b < \infty$
$\text{Poisson}(\theta)$	Poisson distribution with mean $\theta, \theta \in (0, \infty)$
$N(\mu, \sigma^2)$	Normal distribution with mean μ and variance $\sigma^2, \mu \in (-\infty, \infty), \sigma^2 \in (0, \infty)$
X_n^2	Central chi-square distribution with n degrees of freedom, n = 1, 2, …
$F_{m, n}$	F distribution with (m, n) degrees of freedom, $m, n = 1, 2, \ldots$
$\Phi(\cdot)$	Distribution function of N(0, 1)
$\|x\|$	Absolute value of x
MLE	Maximum Likelihood Estimator
$n!$	$n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1, n = 1, 2, 3, \ldots,$ and $0! = 1$
$\binom{n}{k}$	$\frac{n!}{k!(n-k)!}, k = 0, 1, 2, \ldots, n$ and $n = 1, 2, \ldots; \binom{0}{0} = 1$
$\max\{a_1, a_2, \ldots, a_n\}$	Maximum of real numbers $a_1, a_2 \ldots, a_n \ (n \geq 2)$
$\min\{a_1, a_2, \ldots, a_n\}$	Minimum of real numbers $a_1, a_2, \ldots, a_n \ (n \geq 2)$
$\ln x$	Natural logarithm of x

MATHEMATICS STATISTICS

SECTION - A

Multiple Choice Questions (MCQ)

Q.1. to Q.10 carry one mark each

Q.1. Let $\{a_n\}$ be a sequence of positive reals such that $\lim_{n\to\infty}(a_n)^{1/n} = \frac{1}{4}$, then the function

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x^2}\right) & x \neq 0 \\ \lim_{n\to\infty} \frac{\log(1+a_n)}{\sin\left(a_n + \frac{\pi}{2}\right)} & x = 0 \end{cases}$$ is

(A) Continuous at 0, not differentiable at 0

(B) Continuous everywhere except at 0

(C) Differentiable at 0

(D) Nowhere differentiable

Q.2. Let a be a non-zero real number. Then $\lim_{x\to a}\frac{1}{x^2-a^2}\int_a^x \sin(t^2)\,dt$ equals

(A) $\frac{1}{2a}\sin(a^2)$ (B) $\frac{1}{2a}\cos(a^2)$ (C) $-\frac{1}{2a}\sin(a^2)$ (D) $-\frac{1}{2a}\cos(a^2)$

Q.3. X has an Exp (mean 1). The random variable Y is defined by the relationship $Y = \sqrt{X}$. Which of the following is the probability density function of Y?

(A) $\frac{e^{-\sqrt{y}}}{2\sqrt{y}}$ (B) $e^{-\sqrt{y}}$ (C) $2ye^{-y^2}$ (D) e^{-y^2}

Q.4. The joint density function

$$f(x,y)=\begin{cases}6x & \text{for } 0<x<1, 0<y<1-x\\ 0, & \text{otherwise}\end{cases}.$$

Calculate the $E(\min(x,y)) =$

(A) 1/6 (B) 1/4 (C) 1/3 (D) 1/2

Q.5. The distribution function for a discrete non-negative integer-valued random variable X is

$$F_X(k)=1-\sum_{n=k}^{\infty}\frac{e^{-1}}{n!}$$

Calculate the mean of X.

(A) e^{-1} (B) 1 (C) 2 (D) e

Q.6. For three events A, B and C, you are given:

$B\subset A,\ \ P(A\cap C)=0.6,\ \ P(B\cap C)=0.2,\ P(C)=0.8$

Calculate $P(B|A\cap C)$.

(A) 1/6 (B) 1/5 (C) 1/4 (D) 1/3

Q.7. You are given the following table of joint probabilities, P(X = x, Y = y)

y \ x	0	1	2
–1	0.2	0.1	0
0	0.1	0.2	0.1
1	0	0.3	0

Calculate Cov(X, Y).

(A) –0.2 (B) –0.1 (C) 0 (D) 0.2

Q.8. X and Y are independent exponentially distributed random variables, each with a mean of 1. Calculate the median of the random variable X + Y.

(A) Less than 1.0 (B) At least 1.0 but less than 1.2

(C) At least 1.2 but less than 1.4 (D) At least 1.6

Q.9. You are given the following joint density function: $f(x, y) = 2e^{-(x+y)},\ 0 < x < y < \infty$

Calculate Cov(X + Y, X – Y)

(A) –2 (B) –1 (C) 0 (D) 1

Q.10. You are given P(A | B) = 0.5, P(B | A) = 0.4, $P(A \cup B) = 0.7$.

Determine $P(A \cap B)$.

(A) 0.05 (B) 0.15 (C) 0.20 (D) 3

Q.11. to Q.30 carry two mark each

Q.11. You are given the joint distribution of X and Y has joint pdf

$$f(x, y) = \begin{cases} 12xy & \text{for } 0 < x < 1, 0 < y < x^2 \\ 0, & \text{otherwise} \end{cases}$$

Calculate the conditional expectation $E\left[Y \middle| X = \frac{1}{2}\right]$.

(A) 1/10 (B) 1/9 (C) 1/8 (D) 1/6

Q.12. Suppose that X and Y are continuous random variables with joint density function

$$f(x, y) = \begin{cases} 3x & \text{for } 0 < x < 1 \text{ and } 1 - x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Calculate $P[Y < X]$

(A) 1/8 (B) 1/4 (C) 3/8 (D) 5/8

Q.13. X and Y are independent and both have a continuous uniform distribution on [0, 1] / U = min{X, Y} and W = max{X, Y}. Calculate the covariance between U and W.

(A) 1/2 (B) 1/4 (C) 1/16 (D) 1/36

Q.14. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from Cauchy distribution with pdf

$$f(x,\theta)=\frac{1}{\pi\left[1+(x-\theta)^2\right]}, -\infty < x < \infty$$

Which of the following statement is true?

(A) $\overline{X}$ is a consistent estimator of θ

(B) Sample median is a consistent estimator of θ

(C) $\sum_{i=1}^{n} X_i$ is sufficient for θ

(D) $\prod_{i=1}^{n} X_i$ is sufficient for θ

Q.15. Let $X_1, X_2, \ldots, X_n$ be a random sample from $U[\theta, \theta+1]$, then $E\left[X_{(n)} - X_{(1)}\right]$,

where $X_{(1)} = \text{Min}\{X_1, X_2, \ldots, X_n\}$; $X_{(n)} = \text{Max}\{X_1, X_2, \ldots, X_n\}$ is given by

(A) $\frac{n+1}{n-1}$ (B) $\frac{1}{n+1}$ (C) $\frac{n-1}{n-1}$ (D) $\frac{n}{n-1}$

Q.16. Let $X_1, X_2, \ldots, X_n$ ($n > 1$) be a random sample from $U(\theta, \theta+1)$. Consider the following statement.

1. $\left(\overline{X}-\frac{1}{2}\right)$ is an unbiased estimate of θ

2. $\frac{X_{(1)}+X_{(n)}}{2}-\frac{1}{2}$ is unbiased estimate of θ

3. $\frac{X_{(1)}+X_{(n)}}{2}-\frac{1}{2}$ MLE for θ

4. Any value of θ in the interval $\left[X_{(n)}-1, X_{(1)}\right]$ is an MLE.

Select the correct statement using code given below

(A) 1, 2 and 3 (B) 2, 3 and 4 (C) 1, 3 and 4 (D) 1, 2, 3 and 4

Q.17. Which of the following statement are correct?

1. $X_i \sim U(0, \theta)$ then $X_{(n)}$ is consistent for θ

2. $X_i \sim U(0, 5\theta)$ then $\frac{X_{(n)}}{5}$ is MLE of θ

3. $X_i \sim U(\theta, \theta+1)$ then $\frac{X_{(n)} + X_{(1)}}{2}$ is unbiased for θ

Select the correct statement using the code given below:

(A) 1 only (B) 1 and 2 only (C) 1, 2 and 3 (D) 3 only

Q.18. The maximum likelihood estimators (mle) of the parameters α and β of the distribution

$f(x; \alpha, \beta) = \beta e^{-\beta(x-\alpha)}, \ \alpha \le x < \infty, \beta > 0$ are respectively

(A) $X_{(n)}$ and $\frac{1}{\overline{X} - X_{(n)}}$

(B) $X_{(1)}$ and $\frac{1}{\overline{X} - X_{(1)}}$

(C) $X_{(n)}$ and $\frac{1}{\overline{X} + X_{(n)}}$

(D) $X_{(1)}$ and $\frac{1}{\overline{X} + X_{(1)}}$

Q.19. If for an estimator T_n of the parameter θ $\lim_{n\to\infty} P\{|T_n - \theta| < \varepsilon\} = 1$, for all $\varepsilon > 0$, then which of the following statement is correct?

(A) T_n is unbiased consistent estimator of θ if $E(T_n) = \theta$

(B) T_n is unbiased consistent estimator of θ if $E(T_n) \neq \theta$

(C) T_n is biased consistent estimator of θ if $E(T_n) = \theta$

(D) T_n is unbiased inconsistent estimator of θ if $E(T_n) = \theta$

Q.20. A sample of size 1 is taken from an exponential distribution with mean θ. To test $H_0 : \theta = 1$ against $H_1 : \theta > 1$ the test rejects H_0 if $X > 2$. The size of the test is:

(A) $1 - e^{-2}$ (B) 1 (C) e^{-2} (D) $1 - \frac{1}{2}e^{-2}$

Q.21. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from the distribution of X with mean μ and variance σ^2.

The test statistic to the test $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 > \sigma_0^2$ is given by $T_n = \frac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{\sigma_0^2}$. Then the null distribution of T_n is:

(A) χ^2_{n-1} if X has normal distribution

(B) χ^2_n if X has normal distribution

(C) T distribution with $(n-1)$ degree of freedom

(D) F distribution if X has normal distribution

Q.22. Find the triple integral of the function $f(x, y, z) = x^2$ over the region V enclosed by the planes $x = 0$, $y = z = 0$ and $x + y + z = 1$

(A) $\frac{17}{120}$ (B) $\frac{12}{170}$ (C) $\frac{15}{120}$ (D) $\frac{19}{120}$

Q.23. The radius of convergence of the power series $\sum_{n=0}^{\infty} a_n z^{n^2}$, where $a_0 = 1$, $a_n = 3^{-n} a_{n-1}$ for $n \in \mathbb{N}$, is

(A) 0 (B) $\sqrt{3}$ (C) 3 (D) ∞

Q.24. $\lim_{n\to\infty} \frac{1}{n^2}\sum_{r=1}^{n} r\, e^{r/n} =$

(A) 0 (B) 1 (C) e (D) $2e$

Q.25. Let A be a 3 × 3 matrix. Suppose that the eigenvalues of A are –1, 0, 1 with respective eigenvectors: $(1, -1, 0)^T$, $(1, 1, -2)^T$, $(1, 1, 1)^T$, then 6A equals

(A) $\begin{pmatrix} -1 & 5 & 2 \\ 5 & -1 & 2 \\ 2 & 2 & 2 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 5 & 3 \\ 5 & 1 & 3 \\ 3 & 3 & 3 \end{pmatrix}$ (D) $\begin{pmatrix} -3 & 9 & 0 \\ 9 & -3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$

Q.26. You are given the following information regarding the distribution of Y and the conditional distribution of X given Y:

$P(Y=-1)=\frac{1}{6}$, $P(Y=0)=\frac{1}{3}$, $P(Y=1)=\frac{1}{2}$,

$P(X=-1 \mid Y=-1)=\frac{1}{3}$, $P(X=1 \mid Y=-1)=\frac{2}{3}$,

$P(X=-1 \mid Y=0)=\frac{1}{2}$, $P(X=1 \mid Y=0)=\frac{1}{2}$,

$P(X=-1 \mid Y=1)=\frac{2}{3}$, $P(X=1 \mid Y=1)=\frac{1}{3}$,

Calculate $P(XY=1)$.

(A) 1/18 (B) 1/9 (C) 1/6 (D) 2/9

Q.27. X has the following probability function: $P[X=k]=\frac{k+1}{6}$, for k = 0, 1, 2.

Y has the following probability function given X

$P[Y=j \mid X=k]=\frac{1}{k+2}$ for $j=0, 1, \ldots, k+1$.

Calculate E[Y], the mean of the marginal distribution of Y.

(A) 1/2 (B) 2/3 (C) 5/6 (D) 7/6

Q.28. The random variable X, which has a "zero-modified" Poisson distribution defined in the following way. Y has a Poisson distribution with mean $\lambda > 0$.

The probability function for X is defined as follows:

$P[X=0]=\frac{P[Y=0]}{2}$ and $P[X=k]=c\times P[Y=k]$ for $k=1, 2, \ldots$

Which of the following is a correct expression for c?

(A) 1/2 (B) $\frac{2-e^{-y}}{2-2e^{-y}}$ (C) $\frac{2-e^{-y}}{2-e^{-y}}$ (D) $\frac{1-2e^{-y}}{2-2e^{-y}}$

Q.29. The random variable X, which has a "zero-modified" Poisson distribution defined in the following way. Y has a Poisson distribution with mean $\lambda > 0$.

The probability function for X is defined as follows:

$P[X=0] = \frac{P[Y=0]}{2}$ and P[X = k] = c × P[Y = k] for k = 1, 2, …

Which of the following is a correct expression for Var[X]?

(A) $c\lambda + c\lambda^2 + c^2\lambda^2$ (B) $c\lambda - c\lambda^2 + c^2\lambda^2$ (C) $c\lambda + c\lambda^2 - c^2\lambda^2$ (D) $c\lambda - c\lambda^2 - c^2\lambda^2$

Q.10. Which of the following statements are true?

I. If events A and B are independent, then events A and B′ are independent.

II. If events A and B are independent, then events A′ and B′ are independent.

III. If events A and B are independent, then events $A \cup B$ and $A' \cup B'$ are independent.

(A) All but I (B) All but II (C) All but III (D) All

SECTION – B

Multiple Select Questions (MSQ)

Q.31. to Q.40 carry two mark each

Q.31. Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(\theta - 0.5, \theta + 0.5)$ distribution, where $\theta \in \mathbb{R}$. If $X_{(1)} = \min\{X_1, X_2, \ldots, X_n\}$ and $X_{(n)} = \max\{X_1, X_2, \ldots, X_n\}$, then which one of the following estimator is a maximum likelihood estimator of θ?

(A) $\frac{1}{2}\left(X_{(1)} + X_{(n)}\right)$ (B) $\frac{1}{4}\left(3X_{(1)} + X_{(n)} + 1\right)$

(C) $\frac{1}{4}\left(X_{(1)} + 3X_{(n)} - 1\right)$ (D) $\frac{1}{2}\left(3X_{(n)} - X_{(1)} - 2\right)$

Q.32. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from uniform distribution on $(\theta, \theta+1)$, where $\theta \in R$ is an unknown parameter. Let $X_{(1)} < X_{(2)} <, \ldots, X_{(n)}$ be the corresponding order-statistics. Which of the following are $100(1-\alpha)\%$ confidence interval for θ ?

(A) $\left(-\infty, X_{(n)} - \alpha^{\frac{1}{n}}\right)$ (B) $\left(X_{(n)} + \alpha^{1/n} - 1, \infty\right)$

(C) $\left(X_{(n)} + \frac{\alpha}{2} - 1,\ X_n - \frac{\alpha}{2}\right)$ (D) $\left(-\infty, X_1 - \alpha\right)$

Q.33. Let $\{X_n; n \geq 1\}$ be i.i.d uniform (–1, 2) random variables. Which of the following statements are true?

(A) $\frac{1}{n}\sum_{i=1}^{n} X_i \to 0$ almost surely (B) $\left\{\frac{1}{2n}\sum_{i=1}^{n} X_{2i} - \frac{1}{2n}\sum_{i=1}^{n} X_{2i-1}\right\} \to 0$

(C) $\sup\{X_1, X_2, \ldots\} = 2$ almost surely (D) $\inf\{X_1, X_2, \ldots\} = -1$ almost surely

Q.34. $\{X_i\}_{i\geq 1}$ be a sequence of i.i.d random variables with $E(X_i) = 0$ and $V(X_i) = 1$. Which of the following are true?

(A) $\frac{1}{n}\sum_{i=1}^{n} X_i^2 \to 0$ in probability (B) $\frac{1}{n^{3/4}}\sum_{i=1}^{n} X_i \to 0$ in probability

(C) $\frac{1}{n^{1/2}}\sum_{i=1}^{n} X_i \to 0$ in probability (D) $\frac{1}{n}\sum_{i=1}^{n} X_i^2 \to 1$ in probability

Q.35. Suppose $A \in M_{3\times 3}(\mathbb{R})$ be a matrix satisfying

- All eigenvalues of A are of unit modulus
- $e^{i\left(\frac{4\pi}{3}\right)}$ is an eigenvalue of A

Then trace (A) can be

(A) $2\cos\left(\frac{4\pi}{3}\right) - 1$ (B) 0 (C) $1 + 2\sin\left(\frac{4\pi}{3}\right)$ (D) $1 + 2\cos\left(\frac{4\pi}{3}\right)$

Q.36. Consider the following sequences

$$a_n = \left(\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)...\left(1+\frac{n}{n}\right)\right)^{1/n}$$

$$b_n = \frac{1}{n}\left(\prod_{k=1}^{n}(n+k)\right)^{1/n} \quad \& \quad c_n = \left(\frac{(n+1)(n+2)...(2n)}{n^n}\right)^{1/n}$$

Further suppose $\lim_{n\to\infty} a_n = \alpha$, $\lim_{n\to\infty} b_n = \beta$ and $\lim_{n\to\infty} c_n = \gamma$ then

(A) $\gamma = \frac{\alpha+\beta}{2}$ (B) $\gamma = \sqrt{\alpha\beta}$ (C) $\gamma = \frac{2}{\frac{1}{\alpha}+\frac{1}{\beta}}$ (D) $1 < \gamma < \infty$

Q.37. Which of the following is/are true ?

(A) If $\sum a_n$ be a convergent series of positive terms then the series $\sum\left(\frac{a_1+a_2+...+a_n}{n}\right)$ is convergent

(B) If $\sum a_n$ be a convergent series of positive terms, then the series $\sum\left(\frac{a_1+a_2+...+a_n}{n}\right)$ is divergent

(C) If $\sum a_n$ be a convergent series of positive terms, then the sequence $\left\{\frac{a_1+a_2+...+a_n}{n}\right\}$ is convergent

(D) If $\sum a_n$ be a convergent series of negative terms, then the sequence $\left\{\frac{a_1+a_2+...+a_n}{n}\right\}$ is convergent

Q.38. Let $f:[0,1]\to[0,1]$ be any continuous function, then which of the following is/are necessarily true?

(A) $\exists\alpha\in[0,1]$ such that $f(\alpha)=\alpha$ (B) $\exists\beta\in[0,1]$ such that $f(\beta)=\beta^2$

(C) $\exists\gamma\in[0,1]$ such that $f(\gamma)=\gamma^3$ (D) $\exists\delta\in[0,1]$ such that $f(\delta)=\delta^4$

Q.39. X is a continuous random variable with density function $f(x) = \frac{1}{2}e^{-|x|}$ for $-\infty < x < \infty$.

What is the correct expression for $P\left[X > a \,\middle|\, |X| > a\right]$ for a real number a ?

(A) 1/2

(B) $\begin{cases} \frac{1}{2} \text{ for } a < 0 \\ 1 - \frac{1}{2}e^{-a} \text{ for } a \geq 0 \end{cases}$

(C) $\begin{cases} 1 - \frac{1}{2}e^{a} \text{ for } a < 0 \\ \frac{1}{2}e^{-a} \text{ for } a \geq 0 \end{cases}$

(D) $\begin{cases} 1 - \frac{1}{2}e^{a} \text{ for } a < 0 \\ \frac{1}{2} \text{ for } a \geq 0 \end{cases}$

Q.40. X and Y are independent exponential random variables, each with a mean of 1. Find the probability P[Y > X + 1].

(A) e^{-1} (B) $\frac{e^{-1}}{2}$ (C) $\frac{e^{-1}}{3}$ (D) $\frac{e^{-1}}{4}$

SECTION - C

Numerical Answer Type (NAT)

Q.41. to Q.50 carry one mark each

Q.41. N_1 has a Poisson distribution with a mean of 50 N_2 has a binomial distribution with $n = 100$ and $p = 0.5$. N_1 and N_2 are independent Apply the normal distribution with integer correction to determine the probability $P\left[|N_1 - N_2| > 3\right]$.

Q.42. X has a standard normal distribution.

The function g is defined as follows: $g(t) = P\left[t < X \leq t+1\right]$ for $-\infty < t < \infty$.

Calculate the maximum value of $g(t)$.

Q.43. You are given the following information about the random variable X. On the region $\{0 < x < 1\} \cup \{1 < x \leq 4\}$ the density function of X is $f(x) = cx$, where c is a constant, and at the point X = 1, P(X = 1) = 0.2, and density and probability is 0 elsewhere.

Calculate $F_X(1.5)$ (F_X is the cumulative distribution function of X).

Q.44. The probability function of a binomial random variable is $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$ for integer $n \geq 1$ and $k = 0, 1, \ldots, n$ and the probability generating function of X is / $P_X(t) = (1 - p + pt)^n = (.3 + .7t)^6$. Calculate $E\left[X^3\right]$.

Q.45. X has an exponential distribution with mean $k > 0$. You are given that $E\left[X \middle| X > k\right] = 2$.

Calculate $E\left[X \middle| X \leq k\right]$.

Q.46. X and Y are random variables which have a joint distribution with joint cdf / $F(x, y) = y(x^2 + xy - y^2)$ for $0 \leq y \leq x \leq 1$.

Calculate the covariance between X and Y.

Q.47. You are given the following cumulative distribution function: $F_X(x) = \begin{cases} 0 & x \leq 1 \\ .5x - .5 & 1 < x < 2 \\ .75 & 2 \leq x \leq 3 \\ .25x & 3 < x \leq 4 \\ 1 & x > 4 \end{cases}$

Calculate E[X].

Q.48. X has an exponential distribution with a mean of 1. Find the conditional expectation $E\left[X \middle| 1 < X \leq 2\right]$.

Q.49. Let $X_1, X_2, \ldots, X_{10}$ be independent and identically distributed normal random variables with mean 0 and variance 2. Then $E\left(\dfrac{X_1^2}{X_1^2 + X_2^2 + \ldots + X_{10}^2}\right)$ is equal to …

Q.50. Let $X_1, X_2, \ldots, X_{10}$ be i.i.d N(0, 1) random variables. If $T = X_1^2 + X_2^2 + \ldots + X_{10}^2$, then $E\left(\dfrac{1}{T}\right)$ equals

……..

Q.51. to Q.60 carry two mark each

Q.51. Let U ~ $F_{5,8}$ and V ~ $F_{8,5}$. If P[U > 3.69] = 0.05, then the value of c such that P[V > c] = 0.95 equals …..(round of two decimal places).

Q.52. $S = \{x \in \mathbf{R} : x^{25} = 1 - x\}$ then $|S| = $ ……………. .

Q.53. Let $T : \mathbb{R}^n \to \mathbb{R}^n$ by $T\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ x_1 \\ \vdots \\ x_{n-1} \end{bmatrix}$ then Trace (T) is ……………… .

Q.54. Find the area of that part of the surface of the cylinder $x^2 + y^2 = 4$ which is cut out by the cylinder $x^2 + z^2 = 4$ is ……………… .

Q.55. Evaluate where $\int_0^9 \{\sqrt{x}\}\, dx$, where $\{x\}$ denotes the fractional part of x ………… .

Q.56. X_1, X_2 and X_3 are independent exponential random variables with means of 1, 2 and 3, respectively. Y is defined to the Y = max{X_1, X_2, X_3}. Determine E[Y].

Q.57. X_1, X_2 and X_3 are the outcomes of three independent tosses of a fair six-sided die.

Y = min{X_1, X_2, X_3}. Calculate E[Y].

Q.58. Discrete random variables X and Y have a joint distribution describe by the following joint probability table:

Y \ X	0	1	2
0	**0**	**0.1**	**0.2**
1	**0.1**	**0.2**	**0**
2	**0.2**	**0.2**	**0**

Calculate the coefficient of correlation between X and Y.

Q.59. A fair die is tossed until the first 6 turns up. A record is kept of each toss until this happens. The sum of all the numbers that have been tossed is calculated, including the first 6 and is denoted X. Calculate E(X). You are given that $\sum_{k=1}^{\infty}\left[k \times t^{k}\right]=\frac{t}{(1-t)^{2}}$ for $0 \leq t<1$.

Q.60. X has the following distribution: P[X = 1] = p, P[X = –1] = 1 – p.

$X_1, X_2, \ldots, X_n$ are independent random variables, each of which has the same distribution as X.

Then random variable Y is defined as follows: $Y=\sum_{k=1}^{n} X_{k}$. You are given $\frac{\operatorname{Var}[Y]}{E[Y]}=2.1$.

Calculate p.

END OF THE QUESTION PAPER

MATHEMATICS STATISTICS : ANSWER KEY

Section-A

(Multiple Choice Questions)(MCQ)

1. (A)	2. (A)	3. (C)	4. (D)	5. (C)
6. (D)	7. (D)	8. (D)	9. (B)	10. (C)
11. (D)	12. (D)	13. (D)	14. (B)	15. (C)
16. (D)	17. (B)	18. (B)	19. (A)	20. (C)
21. (A)	22. (A)	23. (B)	24. (B)	25. (A)
26. (D)	27. (D)	28. (B)	29. (C)	30. (C)

Section-B

(Multiple Select Questions)(MSQ)

31. (A, B, C)	32. (A, B, C, D)	33. (A, B, C, D)	34. (B, D)	35. (A, D)
36. (A, B, C, D)	37. (B, C, D)	38. (A, B, C, D)	39. (D)	40. (B)

Section-C

(Numerical Answers Type)(NAT)

41. (0.69)	42. (0.38)	43. (0.31)	44. (89.46)	45. (0.42)
46. (0.04)	47. (2.12)	48. (1.42)	49. (0.125)	50. (0.1)
51. (0.27)	52. (1)	53. (0)	54. (32)	55. (5)
56. (3.93)	57. (2.04)	58. (–0.67)	59. (21)	60. (0.7)