IIT JAM - MATHEMATICS

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections **A, B** and **C**. All sections are compulsory. Question in each section are of different types.

2. **Section - A** contains a total of 30 **Multiple Choice Questions (MCQs)**. Each MCQ type question has four choices out of which only **one** choice is the correct answer. Question Q.1 - Q.30 belong to this section and carry a total of 50 marks. Q.1 - Q.10 carry 1 mark each and Questions Q.11 - Q. 30 carry 2 marks each.

3. **Section - B** contains a total of **10 Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 - Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section - C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 - Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Question Q.51 - Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section - A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks question, 2/3 marks will be deducted for each wrong answer. In **Section - B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section - C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

Special Instructions/Useful Data

$\mathbb{N}$ = the set of all positive integers.

$\mathbb{Z}$ = the set of all integers.

$\mathbb{Q}$ = the set of rational numbers.

$\mathbb{R}$ = the set of real numbers.

$\mathbb{R}^n$ = then n-dimensional Euclidean space.

$\mathbb{C}$ = the set of complex numbers.

$M_n(\mathbb{R})$ = the real vector space of all $n \times n$ matrices with entries in $\mathbb{R}$.

$M_n(\mathbb{C})$ = the complex vector space of all $n \times n$ matrices with entries in $\mathbb{C}$.

$gcd(m, n)$ = the greatest common divisor of the integers m and n.

M^T = the transpose of the matrix M.

A – B = the complement of the set B in the set A, that is, $\{x \in A : x \notin B\}$.

$\ln x$ = the natural logarithm of x (to the base e).

$|x|$ = the absolute value of x.

y', y'', y''' = the first, second and the third derivatives of the function y, respectively.

S_n = the symmetric group consisting of all permutations of $\{1, 2, ..., n\}$.

$\mathbb{Z}_n$ = the additive group of integers modulo n.

$f \circ g$ is the composite function defined by $(f \circ g)(x) = f(g(x))$.

The phrase **'real vector space'** refers to a vector space over $\mathbb{R}$.

MATHEMATICS

SECTION - A

Multiple Choice Questions (MCQs):

Q.1. to Q.10 carry one mark each

Q.1. Which of the following groups has a proper subgroup that is NOT cyclic?

(A) $\mathbb{Z}_{15} \times \mathbb{Z}_{77}$ (B) S_3 (C) $(\mathbb{Z}, +)$ (D) $(\mathbb{Q}, +)$

Q.2. Let f, g : $[0, 1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x & \text{if } \; x = \dfrac{1}{n} \text{ for } n \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases}$$

and $$g(x) = \begin{cases} 1 & \text{if } \; x \in \mathbb{Q} \cap [0,1] \\ 0 & \text{otherwise} \end{cases}$$

Then

(A) Both f and g are Riemann integrable (B) f is Riemann integrable, g is not

(C) g is Riemann integrable, f is not (D) Neither f nor g is Riemann integrable

Q.3. Let $f : \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right) \to \mathbb{R}$ be given by $f(x) = \dfrac{\pi}{2} + x - \tan^{-1} x$. Consider the following statements:

P: $|f(x) - f(y)| < |x - y|$ for all $x, y \in \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$.

Q: f has a fixed point.

Then

(A) Both P and Q are TRUE (B) P is TRUE and Q is FALSE

(C) P is FALSE and Q is TRUE (D) Both P and Q are FALSE

Q.4. Find the real number α such that the differential equation

$\dfrac{d^2y}{dx^2} + 2(\alpha - 1)(\alpha - 3)\dfrac{dy}{dx} + (\alpha - 2)y = 0$ has a solution $y(x) = a\cos(\beta x) + b\sin(\beta x)$ for some

non zero reals a, b and β.

(A) $\alpha = 1$ (B) $\alpha = 2$ (C) $\alpha = 3$ (D) $\alpha = 4$

Q.5. If $a + b + c = 0$, then

(A) $3ax^2 + 2bx + c = 0$ for at least one $x \in (0, 1)$

(B) $3ax^2 + 2bx + c = 0$ for at least one $x \in (1, 2)$

(C) $3ax^2 + 2bx + c = 0$ for at least one $x \in (-1, 0)$

(D) $3ax^2 + 2bx + c \neq 0$ for any $x \in \mathbb{R}$

Q.6. Let f be bounded on $[a, b]$ and p be any partition of $[a, b]$, then

(A) $\int_{\underline{a}}^{b} f \leq \text{L}(f, p) \leq \text{U}(f, p) \leq \int_{a}^{\overline{b}} f$

(B) $\text{L}(f, p) \leq \int_{\underline{a}}^{b} f \leq \int_{a}^{\overline{b}} f \leq U(f, p)$

(C) For any partition p^* such that $p \subseteq p^*$ we have $\text{U}(f, p) \leq \text{U}(f, p^*)$

(D) For any partition p^* such that $p \subseteq p^*$ we have $\text{L}(f, p^*) \leq \text{L}(f, p) \leq \text{U}(f, p) \leq \text{U}(f, p^*)$

Q.7. If [.] is the G.I.F. then $\int_{-2}^{2} [x^2]dx$

(A) $\sqrt{5} - \sqrt{3} - \sqrt{2}$ (B) $5 - \sqrt{3} - \sqrt{2}$ (C) $\sqrt{5} - \sqrt{3} - 2$ (D) $\sqrt{5} - 3 - \sqrt{2}$

Q.8. Find out the false statement:

(A) f is $\uparrow$, g is $\uparrow$ $\Rightarrow$ $f + g$ is $\uparrow$

(B) f is $\downarrow$, g is $\uparrow$ $\Rightarrow$ $g \circ f$ is $\downarrow$

(C) f is $\downarrow$, g is $\downarrow$ $\Rightarrow$ $g \circ f$ is $\uparrow$

(D) f is $\downarrow$, g is $\downarrow$ $\Rightarrow$ fg is $\uparrow$

Q.9. Suppose $(a_n)_{n\geq 1}$ and $(b_n)_{n\geq 1}$ are two bounded sequences of real numbers.

Which of the following is true?

(A) $\lim_{n\to\infty} \sup\left(a_n + (-1)^n b_n\right) = \lim_{n\to\infty} \sup a_n + \left|\lim_{n\to\infty} \sup b_n\right|$

(B) $\lim_{n\to\infty} \sup\left(a_n + (-1)^n b_n\right) \leq \lim_{n\to\infty} \sup a_n + \lim_{n\to\infty} \sup b_n$

(C) $\lim_{n\to\infty} \sup\left(a_n + (-1)^n b_n\right) \le \lim_{n\to\infty} \sup a_n + \left|\lim_{n\to\infty} \sup b_n\right| + \left|\lim_{n\to\infty} \inf b_n\right|$

(D) $\lim_{n\to\infty} \sup\left(a_n + (-1)^n b_n\right)$ may not exist

Q.10. Suppose A and B are similar real matrices, that is, there exists an invertible matrix S such that $A = SBS^{-1}$.

Which of the following need not be true?

(A) Transpose of A is similar to the transpose of B

(B) The minimal polynomial of A is same as the minimal polynomial of B

(C) Trace (A) = Trace (B)

(D) Eigenvectors of A and Eigenvectors of B are same

Q.11- Q.30 Carry Two Marks Each

Q.11. Let A be an invertible 5 × 5 matrix over a field F. Suppose that characteristic polynomials of A and A^{-1} are the same.

Which of the following is necessarily true?

(A) $\det(A)^2 = 1$ (B) $\det(A)^5 = 1$ (C) $\operatorname{trace}(A)^2 = 1$ (D) $\operatorname{trace}(A)^5 = 1$

Q.12. Let G be a simple group of order 168. How many elements of order 7 does it have?

(A) 6 (B) 7 (C) 48 (D) 56

Q.13. The value of $\int_0^{4\pi} |\sin x|\, dx$ is

(A) 8 (B) –8 (C) 2 (D) –2

Q.14. The quotient group $\mathbb{Q}(8)/\{1,-1\}$ is isomorphic to

(A) $(\mathbb{Q}(8),\cdot)$ (B) $(\{1,-1\},\cdot)$ (C) $(V_4, +)$ (D) $(\mathbb{Z}_4,+)$

Q.15. Let V denote the vector space $C^5[a,b]$ over $\mathbb{R}$ and W = $\left\{f \in V : \frac{d^4f}{dt^4} + 2\frac{d^2f}{dt^2} - f = 0\right\}$. Then

(A) dim(V) = ∞ and dim(W) = ∞ (B) dim(V) = ∞ and dim(W) = 4

(C) dim(V) = 6 and dim(W) = 5 (D) dim(V) = 5 and dim(W) = 4

Q.16. Which one of the following groups is simple?

(A) S_3 (B) GL(2, R) (C) $Z_2 \times Z_2$ (D) A_5

Q.17. Let $\omega = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$, M = $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$, N = $\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$ and G = $\langle M, N \rangle$ be the group generated by the matrices M and N under matrix multiplication. If C_n be the cyclic group of order n, up to isomorphism, then

(A) $G/Z(G) \cong C_6$ (B) $G/Z(G) \cong S_3$ (C) $G/Z(G) \cong C_2$ (D) $G/Z(G) \cong C_4$

Q.18. Let $f(x) = x^5 - 5x + 2$. Then

(A) f has no real root (B) f has exactly one real root

(C) f has exactly three real roots (D) All roots of f are real

Q.19. A particular solution of $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + \frac{y}{4} = \frac{1}{\sqrt{x}}$ is

(A) $\frac{1}{2\sqrt{x}}$ (B) $\frac{\log x}{2\sqrt{x}}$ (C) $\frac{(\log x)^2}{2\sqrt{x}}$ (D) $\frac{(\log x)\sqrt{x}}{2}$

Q.20. Consider the improper Riemann integral $\int_0^x y^{-1/2}\,dy$. This integral is:

(A) Continuous in $[0, \infty)$ (B) Continuous only in $(0, \infty)$

(C) Discontinuous in $(0, \infty)$ (D) Discontinuous only in $\left(\frac{1}{2}, \infty\right)$

Q.21. Let T : V $\to$ W be a nonsingular linear transformation of a finite dimensional vector space V to any vector space W. Which of the following is true?

(A) dim V < Rank of T (B) dim V = Rank of T

(C) T maps a basis of V to a basis of W (D) dim V > Rank of T

Q.22. Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be the linear map defined by T(x, y) = ($2x - 7y$, $4x + 3y$).

The matrix of T with respect to the ordered basis B = {(1, 3), (2, 5)} is

(A) $\begin{bmatrix} 121 & 201 \\ -70 & -116 \end{bmatrix}$ (B) $\begin{bmatrix} -121 & 201 \\ 70 & -116 \end{bmatrix}$ (C) $\begin{bmatrix} -121 & 201 \\ 70 & 116 \end{bmatrix}$ (D) $\begin{bmatrix} -121 & -201 \\ 70 & 116 \end{bmatrix}$

Q.23. Let $f:[0,1]\to\mathbb{R}$ be defined by

$$f(x)=\begin{cases}1/2 & \text{if } x=1/4\\ 1/4 & \text{if } x=1/2\\ 0 & \text{if } x\in[0,1]\setminus\{1/4,1/2\}\end{cases}$$

Then

(A) f is Riemann integrable and $\int_0^1 f(x)\,dx=3/4$

(B) f is Riemann integrable and $\int_0^1 f(x)\,dx=1/4$

(C) f is Riemann integrable and $\int_0^1 f(x)\,dx=0$

(D) f is not Riemann integrable.

Q.24. Let $T:\mathbb{R}^3\to\mathbb{R}^3$ be defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 - x_2, 0)$.

If N(T) and R(T) denote the null space and the range space of T respectively, then

(A) dim N(T) = 2 (B) dim R(T) = 2 (C) R(T) = N(T) (D) $N(T) \subset R(T)$

Q.25. Let $\{x_n\}$ b a sequence of positive numbers such that $\lim_{n\to\infty} x_n = x$, define

$$z_n=\frac{1}{n}\left[x_1\left(1+\frac{x}{n}\right)^n+x_2\left(1+\frac{x}{n-1}\right)^{n-1}+\ldots+x_n(1+x)\right]$$

Then

(A) $\{z_n\}$ converges to e^x (B) $\{z_n\}$ converges to xe^x

(C) $\{z_n\}$ does not have a limit (D) $\{z_n\}$ converges to xe

Q. 26. Which one of the following statements is true?

(A) Exactly half of the elements in any even order subgroup of S_5 must be even permutations

(B) Any abelian subgroup of S_5 is trivial

(C) There exists a cyclic subgroup of S_5 of order 6

(D) There exists a normal subgroup of S_5 of index 7

Q.27. Let A be a 3 × 3 matrix and suppose that the matrix B is obtained from A by the following row operations

$R_1 \leftrightarrow R_2;\ R_3 \to R_3 + 5R_2$. If det(A) = 8, what is det(B) ?

(A) 8 (B) –40 (C) 40 (D) –8

Q.28. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \cot\frac{y}{x}$ where c is a constant, is

(A) cosec(y/x) = c/x (B) cosec(y/x) = cx (C) sec(y/x) = cx (D) sec(y/x) = c/x

Q.29. Consider the following statements

P: The differential equation $\frac{dy}{dx} = y$, $y(0) = 0$ has a unique solution

Q: Let $y(x)$ be a real valued function defined on $\mathbb{R}$ such that $y' = y(1-y)$ with $y(0) \in [0,1]$ then $\lim_{x\to\infty} y(x) = 1$

Then the correct option is

(A) P is true & Q is true (B) P is true but Q is false

(C) P is false but Q is true (D) Neither P nor Q is true

Q.30. Let y be the solution of $y' + y = |x| \quad x \in \mathbb{R}$, $y(-1) = 0$, then $y(1)$ equals

(A) $\frac{2}{e} - \frac{2}{e^2}$ (B) $\frac{2}{e} - 2e^2$ (C) $2 - \frac{2}{e}$ (D) $2 - 2e$

SECTION - B

Multiple Select Questions

Q.31. to Q.40 carry Two marks each

Q.31. Let $\sum_{n=1}^{\infty} a_n x^n$ converges at $x = -3$ but diverges at $x = -4$ & $\sum_{n=1}^{\infty} b_n x^n$ converges at $x = 3$ but diverges at $x = 4$, then

(A) $\sum_{n=1}^{\infty} a_n$ converges (B) $\sum_{n=1}^{\infty} (-1)^n a_n$ converges

(C) $\sum_{n=1}^{\infty}(-1)^n b_n$ converges (D) $\sum_{n=1}^{\infty} b_n$ converges

Q.32. For $z = x + iy \in \mathbb{C}$ define $f : \mathbb{C} \to \mathbb{C}$ by $f(z) = \sqrt{|\text{Re}(z)||\text{Im}(z)|}$, then

(A) $\lim_{z\to 0} f(z) = 0$ (B) f is differentiable at 0

(C) f is continuous at 0 (D) f is not differentiable at 0

Q.33. Consider the functions f and g given by $f(x) = x^5 + 2x^3 + 1$ & $g(x) = x^3 + \sin(x) + 1$ then

(A) f is onto (B) $g(\alpha) = 0$ for some $\alpha \in \mathbb{R}$

(C) $f(\alpha) = f(\beta)$ for some $\alpha \neq \beta$ in $\mathbb{R}$ (D) $\lim_{x\to 0}\frac{g(x)-1}{x}$ is finite & non zero

Q.34. Which of the following is/are correct

(A) $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = x^6 - 2x^2y - x^4y + 2y^2$ has a saddle point at (0, 0)

(B) $g : \mathbb{R}^2 \to \mathbb{R}^2$ given by $g(x, y) = (e^x \cos y, e^x \sin y)$ is injective

(C) Suppose $h : \mathbb{R}^2 \to \mathbb{R}$ be a function such that $|h(x,y)| \le x^2 + y^2 \quad \forall (x,y) \in \mathbb{R}^2$, then h is continuous at (0, 0)

(D) Suppose $p : \mathbb{R}^2 \to \mathbb{R}$ is defined by $p(x, y) = \begin{cases} |x| & x \neq 0 \\ |y| & x = 0 \end{cases}$, then $p(x, y) = 0$ if & only if $x = y = 0$

Q.35. Which of the given sequences (a_n) satisfy the following identity ?

$\lim_{n\to\infty} \sup a_n = -\lim_{n\to\infty} \inf a_n$

(A) $a_n = 1/n$ for n (B) $a_n = (-1)^n (1 + 1/n)$ for all n

(C) $a_n = 1 + \frac{(-1)^n}{n}$ for all n (D) $a_n = \left(e^n + \pi^n\right)^{\frac{1}{n}}$

Q.36. Consider the statement "$\exists\, \alpha \in (0,1)$ such that $|a_{n+2} - a_{n+1}| \le \alpha |a_{n+1} - a_n| \quad \forall n$ then a possible sequence $\{a_n\}$ is/are given by

(a) $a_{n+1} = \frac{1}{7}\left(a_n^{\ 3} + 2\right)$, $a_1 = \frac{1}{2}$

(b) $a_{n+2} = \frac{1}{2}\left(a_n + a_{n+1}\right)$, $0 < a_1 < a_2$

(c) $a_n = 1 + r + r^2 \ldots\ldots\ldots + r^n$, $0 < r < 1$

(d) $a_{n+1} = \left(1 - \frac{1}{2n}\right)x_n + \frac{1}{2n}x_{n-1}$, $x_1 = \frac{1}{2}$

Q.37. For $t \in \mathbb{R}$, let [t] denote the greatest integer less than or equal to t.

Define function $h : \mathbb{R}^2 \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ by $h(x, y) = \begin{cases} \dfrac{-1}{x^2 - y} & \text{if } x^2 \neq y \\ 0 & \text{if } x^2 = y \end{cases}$

and $g(x) = \begin{cases} \dfrac{\sin x}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Then which of the following is/are TRUE?

(A) $\lim\limits_{(x,y)\to(\sqrt{2},\pi)} \cos\left(\dfrac{x^2 y}{x^2+1}\right) = \dfrac{-1}{2}$

(B) $\lim\limits_{(x,y)\to(\sqrt{2},2)} e^{h(x,y)} = 0$

(C) $\lim\limits_{(x,y)\to(e,e)} \ln(x^{y-[y]}) = e - 2$

(D) $\lim\limits_{(x,y)\to(0,0)} e^{2y} g(x) = 1$

Q.38. Consider the four functions from $\mathbb{R}$ to $\mathbb{R}$

$f_1(x) = x^4 + 3x^3 + 7x + 1$,

$f_2(x) = x^3 + 3x^2 + 4x$,

$f_3(x) = \arctan(x)$ and

$f_4(x) = \begin{cases} x & \text{if } x \notin \mathbb{Z} \\ 0 & \text{if } x \in \mathbb{Z} \end{cases}$

Which of the following statements are correct ?

(A) The range of f_1 is a proper subset of range of f_2

(B) The range of f_2 is a proper subset of range of f_3

(C) The range of f_4 is not an interval

(D) f_2 is onto

Q.39. Let y : $\mathbb{R} \to \mathbb{R}$ be solution of the ordinary differential equation,

$2y'' + 3y' + y = e^{-3x}$, $x \in \mathbb{R}$ satisfying $\lim_{x\to\infty} e^{x} y(x) = 0$. Then

(A) $\lim_{x\to\infty} e^{2x} y(x) = 0$ (B) $y(0) = 1/10$

(C) y is a bounded function on $\mathbb{R}$ (D) $y(1) = 0$

Q.40. For each integer n, define $f_n(x) = x + n$, $x \in \mathbb{R}$ and let G = $\{f_n \mid n \in \mathbb{Z}\}$.

Then

(A) G is a cyclic group under composition

(B) G is a non cyclic group under composition

(C) G is not a group under composition

(D) G is a non abelian group under composition

SECTION - C

Numerical Answer Type Questions

Q.41. to Q.50 carry One mark each

Q.41. The coefficient of $(x-1)^5$ in the Taylor expansion about $x = 1$ of the function

F(x) = $\int_1^x \frac{\log_e t}{t+1}\,dt$, $0 < x < 2$ is .... (correct up to two decimal places)

Q.42. Let $u_n = \frac{n!}{1.3.5...(2n-1)}$, Then $\lim_{n\to\infty} u_n$ is equal to ....

Q.43. The maximum value of $f(x, y) = 49 - x^2 - y^2$ on the line $x + 3y = 10$ is ....

Q.44. Volume of solid that lies under the cone $z^2 = x^2 + y^2$ above xy plane and inside the cylinder $x^2 + y^2 = 2x$ is ....

Q.45. Volume bounded by the paraboloid $x^2 + y^2 = 1 + z$ and $z = 0$ is ....

Q.46. Let M be a 3 × 3 real matrix such that $M^2 = 2M + 3I$. If the determinant of M is –9, then the trace of M equals ....

Q.47. Let $T:\mathbb{R}^4 \to \mathbb{R}^4$ be a linear map such that the null space of T is

$\{(x,y,z,w)\in \mathbb{R}^4 : x+y+z+w=0\}$ and the rank of $(T - 4\mathbf{I}_4)$ is 3.

If the minimal polynomial of T is $x(x-4)^{\alpha}$, then α is equal to ....

Q.48. Consider the vector space V = $\{a_0 + a_1x + a_2x^2 : a_i \in \mathbb{R} \ \ for \ \ i = 0, 1, 2,\}$ of polynomials of degree at most 2. Let $f : V \to \mathbb{R}$ be a linear functional such that $f(1 + x) = 0$, $f(1 - x^2) = 0$ and $f(x^2 - x) = 2$. Then $f(1 + x + x^2)$ equals ....

Q.49. Consider the expansion of the function $f(x) = \dfrac{3}{(1-x)(1+2x)}$ in powers of x, that is valid in $|x| < \dfrac{1}{2}$. Then the coefficient of x^4 is ....

Q.50. If f is an odd function, then the value of integral

$$\int_{-\pi}^{\pi} \frac{f(\sin x)}{f(\cos x) + f(\sin^2 x)}\, dx$$ is ....

<u>Q.51. to Q.60 carry two mark each</u>

Q.51. The number of distinct subgroups of $\mathbb{Z}_{999}$ is ....

Q.52. The number of subgroup of $\mathbb{Z}_7 \times \mathbb{Z}_7$ of order 7 is ....

Q. 53. The number of elements of order 12 in the symmetric group S_7 is equal to ....

Q.54. The sum of the series $\sum_{n=1}^{\infty} \dfrac{1}{(4n-3)(4n+1)}$ is equal to ....

(Rounded off to two decimal places)

Q.55. If $f : [0, \infty) \to \mathbb{R}$ and $g : [0, \infty) \to [0, \infty)$ are continuous functions such that $\int_0^{x^3+x^2} f(t)\, dt = x^2$ and $\int_0^{g(x)} t^2 dt = 9(x+1)^3$ for all $x \in [0, \infty)$, then the value of $f(2) + g(2) + 16\, f(12)$ is equal to .... (Rounded off to two decimal places)

Q.56. Let $\alpha = \int\limits_{x=0}^{4} \int\limits_{y=\sqrt{4-x}}^{2} e^{y^3}\, dy\, dx$, then the value of $3\alpha - e^8$ is equal to ....

Q.57. Consider the differential equation $\frac{dy}{dx} + 2022y = f(x)$, x > 0, where $f(x)$ is continuous function such that that $\lim\limits_{x\to-\infty} f(x) = 2022$, then $\lim\limits_{x\to\infty} y(x) =$ ....

Q.58. Let S = $\{(x, y) \in \mathbb{R}^2 : 0 \le x \le \pi$, min{sin x, cos x} $\le y \le$ max{sin x, cos x}$\}$.

If α is the area of S, then the value of $2\sqrt{2}\,\alpha$ is equal to ....

Q.59. The number of real roots of the polynomial $f(x) = x^{11} - 13x + 5$ is ....

Q.60. Let $\phi : (-1, 1) \to \mathbb{R}$ be defined by $\phi(x) = \int\limits_{x^7}^{x^4} \frac{1}{1+t^3}\, dt$.

If $\alpha = \lim\limits_{x\to 0} \frac{\phi(x)}{e^{2x^4} - 1}$, then 42α is equal to ....

END OF THE QUESTION PAPER

MATHEMATICS : ANSWER KEY

Section-A (Multiple Choice Questions)(MCQ)				
1. (D)	**2. (B)**	**3. (D)**	**4. (C)**	**5. (A)**
6. (B)	**7. (B)**	**8. (D)**	**9. (A)**	**10. (D)**
11. (A)	**12. (C)**	**13. (A)**	**14. (C)**	**15. (B)**
16. (D)	**17. (B)**	**18. (C)**	**19. (C)**	**20. (A)**
21. (B)	**22. (A)**	**23. (C)**	**24. (A)**	**25. (B)**
26. (C)	**27. (D)**	**28. (C)**	**29. (B)**	**30. (A)**
Section-B (Multiple Selection Questions)(MSQ)				
31. (A, B, C, D)	**32. (A, C, D)**	**33. (A, B, D)**	**34. (A, C, D)**	**35. (A, B)**
36. (A, B, C, D)	**37. (A, C, D)**	**38. (A, C, D)**	**39. (A, B)**	**40. (A, B, D)**
Section-C (Numerical Answers Type)(NAT)				
41. (0.04)	**42. (0)**	**43. (39)**	**44. (3.55)**	**45. (1.57)**
46. (5)	**47. (1)**	**48. (1)**	**49. (33 to 33)**	**50. (0)**
51. (8)	**52. (8)**	**53. (420 to 420)**	**54. (0.25)**	**55. (13.4)**
56. (–1)	**57. (1)**	**58. (8)**	**59. (3)**	**60. (21)**