M.A./M.Sc. Entrance Examination 2018

Note: All symbols carry their usual meaning, unless specified otherwise.

1. If $n > 2$, then $n^5 - 5n^3 + 4n$ is divisible by

   (A) 80.

   (B) 100.

   (C) 120.

   (D) 125.

2. If φ is Euler's Phi function then the value of $\varphi(720)$ is

   (A) 144.

   (B) 192.

   (C) 72.

   (D) 248.

3. If $4x \equiv 2 (\text{mod } 6)$ and $3x \equiv 5 (\text{mod } 8)$ then one of the value of x is

   (A) 34.

   (B) 23.

   (C) 26.

   (D) 32.

4. If $A = \begin{bmatrix} a & 2 \\ 1 & b \end{bmatrix}$ is a matrix with eigen values $\sqrt{6}$ and $-\sqrt{6}$, then the values of a and b are respectively,

   (A) 2 and 1.

   (B) -2 and 1.

   (C) 2 and -1.

   (D) 2 and -2.

5. Which one of the statements is false?

   (A) Every quotient group of a cyclic group is cyclic.

   (B) If G is a group and $Z(G)$ is its centre such that the quotient group of G by $Z(G)$ is cyclic, then G is abelian.

(C) If G and H are groups and $f : G \to H$ is a homomorphism then f induces an isomorphism of $\frac{G}{\mathrm{Ker}(f)}$ with H.

(D) Every quotient group of an abelian group is abelian.

6. Let G be an abelian group of order 2018 and $f : G \to G$ be defined as $f(x) = x^5$. Then

(A) f is an automorphism of G.

(B) f is not injective.

(C) f is not surjective.

(D) there exists $e \neq x \in G$ such that $f(x) = x^{-1}$.

7. Let $G = \{a_1, a_2, \ldots, a_{25}\}$ be a group of order 25. For $b, c \in G$ let

$$bG = \{ba_1, ba_2, \ldots, ba_{25}\},\ Gc = \{a_1c, a_2c, \ldots, a_{25}c\}.$$

Then

(A) $bG = Gc\ \forall b, c \in G$.

(B) $bG \neq Gc$, if $b \neq c$.

(C) $bG = Gc$ only if $b = c$.

(D) $bG = Gc$ only if $b^{-1} = c$.

8. Let $\sigma = (37125)(43216) \in S_7$, the symmetric group of degree 7. The order of σ is

(A) 2.

(B) 4.

(C) 5.

(D) 7.

9. Let p be a prime and let G be a non-abelian p-group. The least value of m such that $p^m \backslash o\left(\frac{G}{Z(G)}\right)$ is

(A) 0.

(B) 1.

(C) 2.

(D) 3.

10. The greatest common divisor of $11 + 7i$ and $18 - i$ in the ring of Gaussian integers $Z[i]$ is

(A) $3i$.

(B) $2 + i$.

(C) $1 + i$.

(D) 1.

11. Which one of the following statements is false?

(A) A subring of the ring of integers Z, is an ideal of Z.

(B) A subring of a field is a subfield.

(C) A field has no proper ideals.

(D) A commutative ring with unity is a field if it has no proper ideals.

12. Let the polynomial $f(x) = 3x^5 + 15x^4 - 20x^3 + 10x + 20 \in Z[x]$, and $f_0(x)$ be the polynomial in $Z_3[x]$ obtained by reducing the coefficients of $f(x)$ modulo 3. Which one of the following statements is true?

(A) $f(x)$ is irreducible over Θ, $f_0(x)$ is irreducible over Z_3.

(B) $f(x)$ is reducible over Θ, $f_0(x)$ is irreducible over Z_3.

(C) $f(x)$ is irreducible over Θ, $f_0(x)$ is reducible over Z_3.

(D) $f(x)$ is reducible over Θ, $f_0(x)$ is reducible over Z_3.

13. Let R be a ring with characteristic n where $n \geq 2$. If M is the ring of 2×2 matrices over R then the characteristic of M is

(A) n.

(B) 0.

(C) $n - 1$.

(D) 1.

14. The dimension of the vector space of all 6×6 real skew-symmetric matrices is

(A) 15.

(B) 21.

(C) 30.

(D) 36.

15. If $S = \{(1, 0, i), (1, 2, 1)\} \subseteq X^3$ then $S^{\perp}$ is

(A) span $\{(-i, \frac{1}{2}(i + 1), 1)\}$.

(B) span $\{(i, -\frac{1}{2}(i + 1), -1)\}$.

(C) span $\{(i, -\frac{1}{2}(i + 1), 1)\}$.

(D) span $\{(i, \frac{1}{2}(i+1), -1)\}$.

16. If $\{x, y\}$ is an orthonormal set in an inner product space then the value of $\|x-y\| + \|x+y\|$ is

(A) 2.

(B) $2\sqrt{2}$.

(C) $\sqrt{2}$.

(D) $2+\sqrt{2}$.

17. Let $f : P^2 \to P$ be defined as $f(x, y) = |x| + |y|$. Then which one of the following statements is true?

(A) f is continuous at (0, 0) but f_x and f_y does not exist at (0, 0).

(B) f is continuous at (0, 0) and $f_x(0,0) = f_y(0,0)$.

(C) f is continuous at (0, 0) and $f_x(0,0) \neq f_y(0,0)$.

(D) f is discontinuous at (0, 0) and $f_x(0,0) = f_y(0,0)$.

18. The rate of change of $f(x, y) = 4y - x^2$ at the point (1, 5) in the direction from (1, 5) to the point (4, 3) is

(A) $\frac{-19}{\sqrt{13}}$.

(B) $\frac{-14}{\sqrt{13}}$.

(C) $\frac{-6}{\sqrt{5}}$.

(D) $\frac{-12}{\sqrt{5}}$.

19. If $f(x) = \lim_{n\to\infty} S_n(x)$, where

$$S_n(x) = \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \cdots + \frac{x}{(nx+1)((n+1)x+1)}$$

then the function f is

(A) continuous everywhere.

(B) continuous everywhere except at countably many points.

(C) continuous everywhere except at one point.

(D) continuous nowhere.

20. Let f be a convex function with $f(0) = 0$. Then the function g defined on $(0, +\infty)$ as $g(x) = \frac{f(x)}{x}$

(A) is an increasing function.

(B) is a decreasing function.

(C) is neither increasing nor decreasing function.

(D) is such that its monotonicity cannot be determined.

21. The equation of the tangent plane to the surface $z = 2x^2 - y^2$ at the point (1, 1, 1) is

(A) $2x - y - 2z = 1$.

(B) $4x - 2y - z = 1$.

(C) $x - y - 2z = 2$.

(D) $4x - y - 3z = 1$.

22. If $f : [0, 10] \to$ P is defined as

$$f(x) = \begin{cases} 0, & 0 \le x < 2, \\ 1, & 2 \le x \le 5 \\ 0, & 5 < x \le 10, \end{cases}$$

and $F(x) = \int_0^x f(t)dt$ then

(A) F is differentiable everywhere on [0, 10].

(B) $F'(x) = f(x)$ for every x.

(C) F is not differentiable at $x = 2$ and $x = 5$.

(D) $F(x) = 3$ for $x \le 5$.

23. The improper integral $\int_{-\infty}^{0} 2^x dx$ is

(A) convergent and converges to –ln2.

(B) convergent and converges to $\frac{1}{\ln 2}$.

(C) divergent.

(D) convergent and converges to 2.

24. The Maclaurin series expansion

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

is valid

(A) if $x > -1$.

(B) only if $x \in (-1,1]$.

(C) only if $x \in [-1,1]$.

(D) for every $x \in$ P.

25. If $a_n = n^{\sin(\frac{n\pi}{2})}$ then

(A) $\limsup a_n = 1, \liminf a_n = -1$.

(B) $\limsup a_n = +\infty, \liminf a_n = -\infty$.

(C) $\limsup a_n = +\infty, \liminf a_n = -1$.

(D) $\limsup a_n = +\infty, \liminf a_n = 0$.

26. If f: P → P is a continuous function such that

$$f(x+y) = f(x) + f(y), \text{ for all } x, y \in \text{P},$$

then

(A) f is a not an increasing function.

(B) f is neither an increasing nor a decreasing function.

(C) f is increasing if $f(1) \geq 0$ and decreasing if $f(1) \leq 0$.

(D) f is increasing if $f(1) \leq 0$ and decreasing if $f(1) \geq 0$.

27. Let *P* be the set of all the polynomials with rational coefficients and *S* be the set of all sequences of natural numbers. Then which one of the following statements is true?

(A) Both the sets *P* and *S* are countable.

(B) *P* is countable but *S* is not.

(C) *S* is countable but *P* is not.

(D) Both the sets *P* and *S* are uncountable.

28. Let

$$S = \bigcap_{n=1}^{\infty} \left[2 - \frac{1}{n}, 3 + \frac{1}{n}\right].$$

Then *S* equals

(A) [2, 3).

(B) (2, 3].

(C) (2, 3).

(D) [2, 3].

29. If $\langle x_n \rangle$ is a sequence such that $x_n \geq 0$, for every $n \in \mathbb{N}$ and if $\lim_{n\to\infty}((-1)^n x_n)$ exists then which one of the following statements is true?

(A) The sequence $\langle x_n \rangle$ is divergent.

(B) The sequence $\langle x_n \rangle$ is a Cauchy sequence.

(C) The sequence $\langle x_n \rangle$ is unbounded.

(D) The sequence $\langle x_n \rangle$ is not a Cauchy sequence.

30. If $\langle x_n \rangle$ is a sequence defined as

$$x_n = \left[\frac{5+n}{2n}\right], \text{ for every } n \in \mathbb{N}$$

where [.] denotes the greatest integer function then $\lim_{n\to\infty} x_n$

(A) does not exist.

(B) $\frac{1}{2}$.

(C) 0.

(D) 1.

31. Let

$$S = \bigcap_{n=1}^{\infty} \left[0, \frac{1}{n}\right].$$

Then which one of the following statements is true?

(A) sup $S > 0$.

(B) sup S = inf $S = 0$.

(C) inf $S > 0$.

(D) sup $S = 1$ and inf $S = 0$.

32. How many continuous real functions f can be defined on $\mathbb{R}$ such that $(f(x))^2 = x^2$ for every $x \in \mathbb{R}$?

(A) 2.

(B) 4.

(C) None.

(D) Infinitely many.

33. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function which takes irrational values at rational points and rational values at irrational points. Then which one of the following statements is true?

(A) f is uniformly continuous on $\mathbb{R}$.

(B) f is uniformly continuous on $\mathbb{Q}$.

(C) f is uniformly continuous on $\mathbb{Q}^c$.

(D) No such function exists.

34. Let A and B be two subsets of a metric space X. If int$A$ denotes the interior A of then which one of the following statements is not true?

(A) $\text{int}(A \cup B) = \text{int}A \cup \text{int}B$.

(B) $\text{int}(A \cup B) \supseteq \text{int}A \cup \text{int}B$.

(C) $\text{int}(A \cap B) = \text{int}A \cap \text{int}B$.

(D) $A \subseteq B \Rightarrow \text{int}A \subseteq \text{int}B$.

35. Which one of the following spaces, with the usual metric, is not separable?

(A) The Euclidean space $\mathbb{R}^n$.

(B) The space $C[a, b]$ of the set of all real valued continuous functions defined on $[a, b]$.

(C) The space l^1 of all absolutely convergent real sequences.

(D) The space l^∞ of all bounded real sequences with supremum metric.

36. Let $(x_0, f(x_0)) = (0, -1), (x_1, f(x_1)) = (1, a)$ and $(x_2, f(x_2)) = (2, b)$. If the first order divided differences $f[x_0, x_1] = 5$ and $f[x_1, x_2] = c$ and the second order divided difference $f[x_0, x_1, x_2] = -\frac{3}{2}$, then the values of a, b and c are

(A) 4, 6, 2.

(B) 2, 4, 6.

(C) 6, 2, 4.

(D) 4, 2, 4.

37. For cubic spline interpolation which one of the following statements is true?

(A) The first derivatives of the splines are continuous at the interior data points but not the second derivatives.

(B) The second derivatives of the splines are continuous at the interior data points but not the first derivatives.

(C) The first and the second derivatives of the splines are continuous at the interior data points.

(D) The third derivatives of the splines are continuous at the interior data points.

38. A bound for the error for the trapezoidal rule for the definite integral $\int_0^1 \frac{1}{1+x} dx$ is

(A) $\frac{2}{25}$.

(B) $\frac{1}{15}$.

(C) $\frac{1}{20}$.

(D) $\frac{1}{6}$.

39. Which one of the following statements is not true for Simpson's 1/3 rule to find approximate value of the definite integral $I = \int_0^1 f(x)dx$?

(A) The function $f(x)$ is approximated by a parabola.

(B) Simpson's 1/3 rule improves trapezoidal rule.

(C) If $y_0 = f(0), y_1 = f(0.5), y_2 = f(1)$, the approximate value of I is $\frac{1}{6}[y_0 + 3y_1 + y_2]$.

(D) The approximating function has odd number of points common with the function $f(x)$.

40. Exact value of the definite integral $\int_a^b f(x)dx$ using Simpson's rule

(A) is given when $f(x)$ is a polynomial of degree 4.

(B) is given when $f(x)$ is a polynomial of degree 3.

(C) is given when $f(x)$ is a polynomial of degree 5.

(D) cannot be given for any polynomial.

41. The total number of arithmetic operations required to find the solution of a system of n linear equations in n unknowns by Gauss elimination method is

(A) $\frac{2}{3}n^3 + \frac{1}{2}n^2 - \frac{5}{6}n$.

(B) $n^3 - \frac{1}{6}n$.

(C) $\frac{2}{3}n^3 + \frac{3}{2}n^2 - \frac{7}{6}n$.

(D) $\frac{1}{3}n^3 + \frac{1}{2}n^2 - \frac{5}{6}n$.

42. The central difference operator δ and backward difference operator ∇ are related as

(A) $\delta = \nabla(1 + \nabla)^{\frac{1}{2}}$.

(B) $\delta = \nabla(1 + \nabla)^{-\frac{1}{2}}$.

(C) $\delta = \nabla(1 - \nabla)^{\frac{1}{2}}$.

(D) $\delta = \nabla(1 - \nabla)^{-\frac{1}{2}}$.

43. The eigenvalues for the Sturm–Liouville problem

$$y'' + \lambda y = 0, 0 \leq x \leq \pi,$$
$$y(0) = 0, y'(\pi) = 0$$

are

(A) $\lambda_n = n\pi, n = 1, 2, \dots ..$

(B) $\lambda_n = n^2, n = 1, 2, \dots ..$

(C) $\lambda_n = n^2\pi^2, n = 1, 2, \dots ..$

(D) $\lambda_n = \frac{(2n-1)^2}{4}, n = 1, 2, \dots ..$

44. For the differential equation

$$x\frac{dy}{dx} + 6y = 3xy^{4/3}$$

consider the following statements:

(i) The given differential equation is a linear equation.

(ii) The differential equation can be reduced to linear equation by the transformation $V = y^{-1/3}$.

(iii) The differential equation can be reduced to linear equation by the transformation $V = x^{-1/3}$.

Which of the above statements are true?

(A) Both (i) and (ii).

(B) Only (i).

(C) Only (ii).

(D) Only (iii).

45. The initial value problem

$$x\frac{dy}{dx} - 2y = 0,$$
$$x > 0, y(0) = 0$$

has

(A) infinitely many solutions.

(B) exactly two solutions.

(C) a unique solution.

(D) no solution.

46. The general solution of the system of the differential equations

$$x_1' = 3x_1 - 2x_2$$
$$x_2' = 2x_1 - 2x_2$$

is given by

(A) $\begin{pmatrix} c_1e^t + 2c_2e^{-2t} \\ c_1e^t + c_2e^{-2t} \end{pmatrix}$.

(B) $\begin{pmatrix} c_1e^{-t} + c_2e^{2t} \\ c_1e^{-t} - c_2e^{2t} \end{pmatrix}$.

(C) $\begin{pmatrix} c_1e^{t} + 2c_2e^{-2t} \\ 2c_1e^{t} + 2c_2e^{-2t} \end{pmatrix}$.

(D) $\begin{pmatrix} c_1e^{-t} + 2c_2e^{2t} \\ 2c_1e^{-t} + c_2e^{2t} \end{pmatrix}$.

47. The complete integral of the partial differential equation $xpq + yq^2 - 1 = 0$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ is

(A) $z + b = 2(ax + y)$.

(B) $z + b = 2(ax + y)^2$.

(C) $(z + b)^2 = 4(ax + y)$.

(D) $z + b = 4(ax + y)^2$.

48. The characteristics of the partial differential equation

$$36\frac{\partial^2 z}{\partial x^2} - y^{14}\frac{\partial^2 z}{\partial y^2} - 8x^{12}\frac{\partial z}{\partial x} = 0$$

when it is of hyperbolic type are given by

(A) $x + \frac{1}{y^6} = c_1, x - \frac{1}{y^6} = c_2$.

(B) $x + \frac{36}{y^6} = c_1, x - \frac{36}{y^6} = c_2$.

(C) $x + \frac{1}{y^7} = c_1, x - \frac{1}{y^7} = c_2$.

(D) $x + \frac{36}{y^7} = c_1, x - \frac{36}{y^7} = c_2$.

49. The complete integral of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+2y}$$

is

(A) $\phi_1(y - x) + \phi_2(y + x) + e^{x+2y}$.

(B) $\phi_1(y + x) + x\phi_2(y + x) + e^{x+2y}$.

(C) $\phi_1(y - x) + x\phi_2(y + x) + e^{x+2y}$.

(D) $\phi_1(y + x) + x\phi_2(y + x) + xe^{x+2y}$.

50. The partial differential equation

$$(x^2 - 1)\frac{\partial^2 z}{\partial x^2} + 2y\frac{\partial^2 z}{\partial x \partial y} - \frac{\partial^2 z}{\partial y^2} = 0$$

is

(A) hyperbolic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$.

(B) parabolic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$.

(C) elliptic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 1\}$.

(D) hyperbolic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 1\}$.