

SEAL

M A / M.Sc

MATHS. UNIT 2015-16

11029

SUBJECT CODE : GS-6030-A

Test Booklet Serial No. :

Series : **A**

Total Number of Pages : 24

TEST BOOKLET

(Read the instructions carefully before starting to answer)

Time : 2 Hours

Maximum Marks : 200

- Fill up the following information by Blue or Black ball point pen only :

Roll No.

Name of the Candidate

Name of Examination Centre

Date of Examination

Candidate's Signature

Signature of Invigilator

- Open the seal of the booklet only when instructed to do so.
- Don't start answering the questions until you are asked to do so.
- Ensure that there are 50 questions in the Test Booklet with four responses (A), (B), (C) and (D). Of them only one is correct as the best answer to the question concerned.
- There will be NEGATIVE MARKING for wrong answer. Each correct answer shall be awarded 4 marks, while one mark will be deducted for each wrong answer.
- Multiple answering of a question will cause the answer to be rejected.
- Use only Black or Blue Ball pen for darkening appropriate circle completely.

For example :



- Rough work is to be done only on the Test Booklet and not on the answer sheet.
- You are not allowed to use Mobile Phones or any Electronic Device. Only Non-Programmable calculator is allowed.
- Make sure that you do not possess any pages (Blank or Printed) or any unauthorized material. If such material is found in your possession during the examination, you will be disqualified for admission.
- If you are found copying/helping others, you will be disqualified for admission.
- At the end of the examination hand over the answer sheet to the invigilator.
- Do not leave the examination hall until you are asked to do so.
- No candidate is allowed to leave the examination hall till the completion of examination.
- The candidates are allowed to take the Test Booklet with them.
- Candidates are advised to contact the Examination Superintendent for submission of representation related to examination, if any.
- Smoking and eatables are not allowed inside the examination hall.

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Note : All symbols carry their usual, unless specified otherwise.

1. The sequence $(n^{1/n})$ is
 - (A) monotonically decreasing
 - (B) monotonically increasing
 - (C) convergent and converges to zero
 - (D) neither monotonically increasing nor monotonically decreasing

2. Let

$$S = \prod_{n=1}^{\infty} \left[-\frac{1}{n}, 1 + \frac{1}{n} \right]$$

then S equals

- (A) $[0, 1]$
 - (B) $(0, 1]$
 - (C) $(0, 1)$
 - (D) $[0, 1)$
3. Consider the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (\sqrt{n+1} - \sqrt{n-1}).$$

Then

- (A) the series is convergent but not absolutely convergent
- (B) the series is divergent
- (C) the n th term of series does not converge to zero
- (D) the series is absolutely convergent

4. Consider the sets

$$S = \left\{ \frac{1}{n} : n \in \mathbb{N} \text{ and } n \text{ is prime} \right\}$$

$$T = \{x^2 : x \in \mathbb{R}\}.$$

Then

- (A) $\sup(S \cap T) = 1$
- (B) $\sup S = 1$ and $\inf T = 0$
- (C) $\sup S = \frac{1}{2}$ and $\inf T = 0$
- (D) $\inf(S \cup T) = \frac{1}{2}$

5. Consider the following functions from $\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$d_1(x, y) = |x| + |y|,$$

$$d_2(x, y) = \begin{cases} 2, & x \neq y \\ 0, & x = y \end{cases}$$

$$d_3(x, y) = \sqrt{|x - y|}.$$

Which of the following statements is true ?

- (A) Only d_2 and d_3 are metrics on \mathbb{R}
- (B) Only d_3 is a metric on \mathbb{R}
- (C) Only d_1 and d_2 are metrics on \mathbb{R}
- (D) All are metrics on \mathbb{R}

6. $S = \{(x, y) \in \mathbb{R}^2 : xy < 0\}$ is

- (A) neither connected nor compact subset of \mathbb{R}^2
- (B) not connected but is compact subset of \mathbb{R}^2
- (C) is both connected and compact subset of \mathbb{R}^2
- (D) is not compact subset of \mathbb{R}^2 but connected

7. Let (x_n) be a sequence defined by :

$$x_1 = 3 \text{ and } x_{n+1} = \frac{1}{4 - x_n}.$$

Then

- (A) (x_n) is a monotonically decreasing sequence that is not bounded below
- (B) (x_n) converges to $2 + \sqrt{3}$
- (C) (x_n) converges to $2 - \sqrt{3}$
- (D) (x_n) diverges

8. The value of the series

$$\sum_{n=1}^{\infty} \frac{n}{2^n}$$

is given by

- (A) 2
- (B) 4
- (C) 6
- (D) 8

9. Let f be a continuous function on \mathbb{R} . Define

$$G(x) = \int_0^{\sin x} f(t) dt \quad \forall x \in \mathbb{R}.$$

Then

- (A) $G'(x) = f(\cos x) \sin x$
- (B) $G'(x) = -f(\sin x) \cos x$
- (C) $G'(x) = f(\sin x) \cos x$
- (D) $G'(x) = f(\sin x) \sin x$

10. Let (X, d) be a metric space where X is an infinite set and d is the discrete metric. Then

- (A) Heine-Borel theorem holds for (X, d)
- (B) Heine-Borel theorem does not hold for (X, d)
- (C) X is not bounded
- (D) X is compact

11. Let

$$f_n(x) = \frac{1}{1 + (nx - 1)^2}, x \in [0, 1].$$

Then the sequence (f_n) is

- (A) pointwise convergent but not uniformly convergent on $[0, 1]$
- (B) uniformly convergent but not pointwise convergent on $[0, 1]$
- (C) both pointwise and uniformly convergent on $[0, 1]$
- (D) neither pointwise nor uniformly convergent on $[0, 1]$

12. The limit inferior of the sequence (x_n) where

$$x_n = 1 + (-1)^n + \frac{1}{3^n}$$

is

13. Which of the following sets is in one-to-one correspondence with \mathbb{N}

- (I) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right\}$

(II) $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

(III) $\left\{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\right\}$

(IV) $\left\{\frac{p}{q} : p, q \in \mathbb{N}\right\}$

- (A) (I) and (II)
 - (B) (I), (II) and (III)
 - (C) (I) and (IV)
 - (D) All of the above

14. Suppose f and g are differentiable on the interval $[a, \infty)$ such that $f(a) \leq g(a)$ and $f'(x) < g'(x) \forall x > a$. Then which of the following statements is true ?

(A) $f(x) = g(x) \forall x \in [a, \infty)$

(B) $f(x) > g(x)$

(C) $f(x) < g(x)$

(D) None of the above

15. Which of the following statements are true ?

(I) There exists a continuous function from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ onto $(0, 1)$

(II) There exists a continuous function from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ onto \mathbb{R}

(III) There exists a continuous function from $[0, \pi] \cup [2\pi, 3\pi]$ onto $[0, 1]$

(IV) There exists a continuous function from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ onto $\left[0, \frac{1}{3}\right] \cup \left[\frac{2}{3}, 1\right]$

(A) (I) and (II)

(B) (II) and (III)

(C) (III) and (IV)

(D) (I) and (IV)

16. For

$$x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in \mathbb{R}^3$$

define

$$d_1(x, y) = \max_{1 \leq j \leq 3} |x_j - y_j|$$

$$d_2(x, y) = \left[\sum_{j=1}^3 (x_j - y_j)^2 \right]^{1/2}.$$

Consider the metric spaces (\mathbb{R}^3, d_1) and (\mathbb{R}^3, d_2) . Then

- (A) (\mathbb{R}^3, d_1) is complete, but (\mathbb{R}^3, d_2) is not complete
- (B) (\mathbb{R}^3, d_2) is complete, but (\mathbb{R}^3, d_1) is not complete
- (C) Both (\mathbb{R}^3, d_1) and (\mathbb{R}^3, d_2) are complete
- (D) Neither (\mathbb{R}^3, d_1) nor (\mathbb{R}^3, d_2) is complete

17. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Then

- (A) f is not continuous at $(0, 0)$ but all directional derivatives of f at $(0, 0)$ exist.
- (B) f is continuous in \mathbb{R}^2 and all directional derivatives of f at $(0, 0)$ exist.
- (C) f is continuous in \mathbb{R}^2 but not all directional derivatives at $(0, 0)$ exist.
- (D) f is not continuous at $(0, 0)$ and not all directional derivatives at $(0, 0)$ exist.

18. Let

$$X = \{(x, y) \in \mathbb{R}^2 : x \in \mathbb{Q}, y \in \mathbb{R} \setminus \mathbb{Q}\}$$

where \mathbb{Q} is the set of rationals. Then

- (A) X is an open and dense subset of \mathbb{R}^2
- (B) X is an open but not dense subset of \mathbb{R}^2
- (C) X is not an open but a dense subset of \mathbb{R}^2
- (D) X is neither an open nor a dense subset of \mathbb{R}^2

19. Let $n \in \mathbb{N}, n \geq 3$ be fixed and let $f : [0, 1] \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x & , \quad 0 \leq x \leq 1/n \\ x - \frac{(2k-1)}{2n} & , \quad \frac{k-1}{n} < x \leq \frac{k}{n} \\ & k = 2, 3, \dots, n. \end{cases}$$

Then

- (A) f is continuous and Riemann integrable on $[0, 1]$.
- (B) f is not continuous but is Riemann integrable on $[0, 1]$.
- (C) f is continuous but not Riemann integrable on $[0, 1]$.
- (D) f is neither continuous nor Riemann integrable on $[0, 1]$.

20. Let

$$S = \{x \in \mathbb{R} : 3 - x^2 > 0\}.$$

Then

- (A) S is bounded above and 3 is the least upper bound of S .
(B) S is bounded above and does not have a least upper bound in \mathbb{R} .
(C) S is bounded above and does not have a least upper bound in \mathbb{Q} , the set of rational numbers.
(D) S is not bounded above.

21. Let p and q be distinct primes and let G and H be two groups such that $o(G) = p$ and $o(H) = q$. The number of distinct homomorphisms from G to H is/are

- (A) 1 (B) $p - 1$
(C) $q - 1$ (D) pq

22. Let G be a cyclic group such that G has an element of infinite order. Then the number of elements of finite order in G is/are

- (A) 0 (B) 1
(C) infinite (D) none of these

23. Let G be a non-abelian group of order p^3 where p is a prime. Let $Z(G) \neq \{e\}$.

Then

- (A) $o(Z(G)) = p$
- (B) $o(Z(G)) = p^2$
- (C) $\frac{G}{Z(G)}$ is cyclic
- (D) none of the above

24. Let G be a group of order pqr , where p, q, r are primes and $p < q < r$. Which of the following statements are true ?

- (i) G has a normal subgroup of order qr
 - (ii) Sylow r -subgroup of G is normal
 - (iii) G is abelian
- (A) only (i) and (ii)
 - (B) only (ii) and (iii)
 - (C) only (i) and (iii)
 - (D) (i), (ii) and (iii)

25. Let R be a ring with unity such that each element of R is an idempotent.

Then the characteristic of R is

- (A) 0
- (B) 2
- (C) an odd prime
- (D) none of the above

26. Let

$$F = \mathbb{Q}(\sqrt{2i}).$$

Which one of the following is *not* true ?

- (A) $\sqrt{2} \in F$
- (B) $i \in F$
- (C) $x^8 - 16 = 0$ has a solution in F
- (D) $\dim_{\mathbb{Q}}(F) = 2$

27. The ideal $\langle x \rangle$ of the ring $\mathbb{Z}[x]$ is

- (A) maximal but not prime
- (B) prime but not maximal
- (C) both prime and maximal
- (D) neither prime nor maximal

28. The smallest subring of \mathbb{Q} containing $\frac{2}{3}$ is

(A) $S = \left\{ a + b \frac{2}{3} \mid a, b \in \mathbb{Z} \right\}$

(B) $S = \mathbb{Q}$

(C) $S = \left\{ a \left(\frac{2}{3} \right)^k \mid k \in \mathbb{N}, a \in \mathbb{Z} \right\}$

(D) $S = \left\{ a_0 + a_1 \frac{2}{3} + a_2 \left(\frac{2}{3} \right)^2 + \dots + a_n \left(\frac{2}{3} \right)^n \mid n \in \mathbb{N}, a_0, a_1, \dots, a_n \in \mathbb{Z} \right\}$

29. If p is an odd prime, then

$$\phi(p) + \phi(2p) + \phi(2^2 p) + \dots + \phi(2^m p)$$

is equal to

(A) $(2^m - 1)(p - 1)$

(B) $2^m(p - 1)$

(C) $(2^m + 1)(p - 1)$

(D) $2^{m+1}(p - 1)$

30. Let

$$A(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \theta \in (0, 2\pi).$$

Which of the following statements is true ?

- (A) $A(\theta)$ has eigenvectors in \mathbb{R}^2 for every $\theta \in (0, 2\pi)$
- (B) $A(\theta)$ does not have eigenvectors in \mathbb{R}^2 for any $\theta \in (0, 2\pi)$
- (C) $A(\theta)$ has eigenvectors in \mathbb{R}^2 for exactly one value of $\theta \in (0, 2\pi)$
- (D) $A(\theta)$ has eigenvectors in \mathbb{R}^2 for exactly two values of $\theta \in (0, 2\pi)$

31. Let $M(n, \mathbb{R})$ be the vector space of $n \times n$ matrices with real entries and U be the subset of $M(n, \mathbb{R})$ given by

$$\{(a_{ij}) \mid a_{11} + a_{22} + \dots + a_{nn} = 0\}.$$

Which one of the following statements is true ?

- (A) U is a subspace of dimension $n^2 - 1$
- (B) U is a subspace of dimension $n^2 - n$
- (C) U is not a subspace
- (D) None of the above

32. Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Then $\det(A^3 - 6A^2 + 5A + 3I)$ is

33. Let

$$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in \mathbb{R} \right\}$$

and

$$W = \left\{ a + bx + cx^2 \mid a, b, c \in \mathbb{R} \right\}.$$

Define $T : V \rightarrow W$ by

$$T \begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a+b) + (b-c)x + (c+d)x^2$$

The null space of T is

- (A) $\left\{ a \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \mid a \in \mathbb{R} \right\}$

(B) $\left\{ a \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \mid a \in \mathbb{R} \right\}$

(C) $\left\{ a \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mid a \in \mathbb{R} \right\}$

(D) $\left\{ a \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \mid a \in \mathbb{R} \right\}$

34. Let

$$W_1 = \{(a, 2a, 0) | a \in \mathbb{R}\},$$

$$W_2 = \{(a, 0, -a) | a \in \mathbb{R}\}.$$

Then

- (A) $W_1 + W_2$ is a subspace of \mathbb{R}^3 but $W_1 \cup W_2$ is not
- (B) $W_1 + W_2$, $W_1 \cup W_2$ are both subspaces of \mathbb{R}^3
- (C) neither $W_1 + W_2$ nor $W_1 \cup W_2$ is a subspace of \mathbb{R}^3
- (D) $W_1 \cup W_2$ is a subspace of \mathbb{R}^3 but $W_1 + W_2$ is not

35. Let $V = C[0, \pi]$ be an inner product space with inner product

$$\langle f, g \rangle = \int_0^\pi f(x) g(x) dx.$$

Let $f(x) = \cos x$, $g(x) = \sin x$. Then

- (A) f, g are orthogonal but not linearly independent
- (B) f, g are orthogonal and linearly independent
- (C) f, g are linearly independent but not orthogonal
- (D) neither f, g are linearly independent nor orthogonal

36. If the partial differential equation

$$(x-2)^2 \frac{\partial^2 u}{\partial x^2} - (y-3)^2 \frac{\partial^2 u}{\partial y^2} + 2x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = u$$

is parabolic in the region $S \subseteq \mathbb{R}^2$ but not in $\mathbb{R}^2 \setminus S$, then S is

- (A) $\{(x, y) \in \mathbb{R}^2 : x = 2 \text{ or } y = 3\}$
- (B) $\{(x, y) \in \mathbb{R}^2 : x = 2 \text{ and } y = 3\}$
- (C) $\{(x, y) \in \mathbb{R}^2 : x = 2\}$
- (D) $\{(x, y) \in \mathbb{R}^2 : y = 3\}$

37. Let $u(x, y)$ be the solution of the Cauchy problem

$$x^2 \frac{\partial u}{\partial x} - y^2 \frac{\partial u}{\partial y} = 0,$$

$$u \rightarrow e^x \text{ as } y \rightarrow \infty.$$

Then $u(1, 1)$

- (A) -1
- (B) 0
- (C) 1
- (D) e^{-2}

38. The initial value problem

$$x \frac{dy}{dx} = 2y,$$

$$y(a) = b$$

has

- (A) infinitely many solutions through $(0, b)$ if $b \neq 0$
- (B) unique solution for all a and b
- (C) no solution if $a = b = 0$
- (D) infinitely many solutions if $a = b = 0$

39. The solution of the differential equation

$$\frac{d^2y}{dx^2} + 4y = \cos 2x,$$

is given by

(A) $c_1 \cos 2x + c_2 \sin 2x + \frac{x}{4} \sin 2x$

(B) $c_1 \cos 2x + c_2 \sin 2x + \frac{x}{2} \sin 2x$

(C) $c_1 \cos 2x + c_2 \sin 2x + \frac{x}{4} \cos 2x$

(D) $c_1 \cos 2x + c_2 \sin 2x + x \cos 2x$

40. The following initial value problem of a first order linear system

$$x' = 3x - 2y, \quad x(0) = 1$$

$$y' = -3x + 4y, \quad y(0) = -2$$

can be converted into an initial value problem of a 2nd order differential equation for $x(t)$. It is

(A) $x'' - 7x' + 6x = 0; \quad x(0) = 1, \quad x'(0) = -2$

(B) $x'' - 7x' + 6x = 0; \quad x(0) = 1, \quad x'(0) = 0$

(C) $x'' - 7x' + 6x = 0; \quad x(0) = 1, \quad x'(0) = 7$

(D) $x'' - x' + 6x = 0; \quad x(0) = 1, \quad x'(0) = -2$

- #### 41. The characteristic values of the Sturm-Liouville problem

$$\frac{d^2y}{dx^2} + \lambda y = 0; \quad y(0) = 0, \quad y'(\pi) = 0,$$

are .

44. What are the solutions α if any, of the equation $x = \sqrt{1+x}$? Does the iteration $x_{n+1} = \sqrt{1+x_n}$ converge to any of these solutions?

(A) Root $= \frac{1+\sqrt{5}}{2}$, iterations converge with $x_0 = 1$

(B) Root $= \frac{1-\sqrt{5}}{2}$, iterations converge with $x_0 = -1$

(C) Both (A) and (B)

(D) Roots $= \frac{1 \pm \sqrt{5}}{2}$ but the iterations do not converge to any root

45. Is the following function a cubic spline on the interval $0 \leq x \leq 2$

$$s(x) = \begin{cases} (x-1)^3 & , \quad 0 \leq x \leq 1 \\ 2(x-1)^3 & , \quad 1 \leq x \leq 2 \end{cases}$$

(A) Yes, it is a cubic spline on $[0, 2]$

(B) It is a cubic spline only on $[0, 1]$

(C) It is a cubic spline only on $[1, 2]$

(D) It is not a cubic spline

46. Consider the second order differential equation

$$x^2 y''(x) + x y'(x) - 9y(x) = 0 \text{ for } x > 0.$$

If the solution satisfies the intial conditions $y(1) = 0$, $y'(1) = 2$, then

$y(2)$ is

(A) $\frac{21}{8}$

(B) $\frac{63}{8}$

(C) $\frac{7}{16}$

(D) $\frac{63}{4}$

47. The eigenvalues associated with the BVP

$$y''(x) - 2y'(x) + (1 - \lambda) y(x) = 0$$

$$y(0) = 0, \quad y(1) = 0$$

is/are

(A) $\lambda = 0$

(B) $\lambda = \pi^2 n^2, \quad n = 1, 2, 3, \dots$

(C) $\lambda = -\pi^2 n^2, \quad n = 1, 2, 3, \dots$

(D) $\lambda = \pi n, \quad n = 1, 2, 3, \dots$

48. The value of

$$I = \int_0^{\sqrt{\pi}} \sin x^2 dx$$

using the trapezium rule with two subintervals is

(A) $\frac{\pi}{4}$

(B) $\frac{\sqrt{\pi}}{4}$

(C) $\frac{\sqrt{\pi}}{2}$

(D) $\frac{\sqrt{2\pi}}{4}$

49. Consider the system of equations

$$\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix},$$

where 'a' is a real constant. Then Gauss-Seidel method for the solution of the above system converges for

- (A) all values of a (B) $|a| < 1$
(C) $|a| > 1$ (D) $a > 2$

50. The error in the value of y at 0.2 when modified Euler's method is used to solve the problem

$$\frac{dy}{dx} = x - y, \quad y(0) = 1, \quad h = 0.2$$

is of the order

- (A) 10^{-1} (B) 10^{-2}
(C) 10^{-3} (D) 0

SPACE FOR ROUGH WORK

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P.T.O.

